



بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

PROBABILITY DISCUSSION

Chapter 1

Getting Stared with Probability 'PART 1'



1.1 Set Theory

- ❖ **Set : a collection of things**
- ❖ **Elements: are things that make up the set**
- ❖ **Set A contain elements x, y, z**

❑ Some definitions:

Inclusion: $x \in A \rightarrow x$ is an element in set A

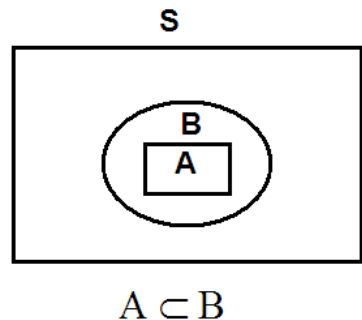
Subset: $A \subset B$

Equality: $A = B \rightarrow A \subset B \ \& \ B \subset A$

Universal Set (S)

Null Set ($\emptyset$) has no elements

Venn diagram : used to display relationships among sets,

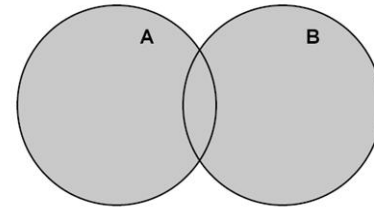


the large rectangle is the universal set S

□ Operations in Set Algebra

(1) Union

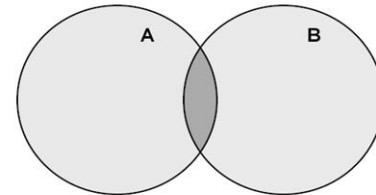
$x \in A \cup B$ iff $x \in A$ or $x \in B$



A or B

(2) Intersection

$x \in A \cap B$ iff $x \in A$ and $x \in B$

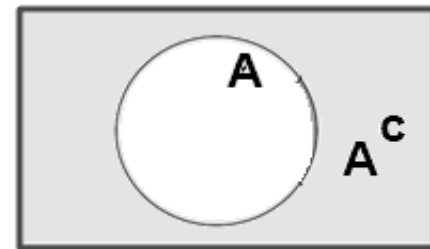


A and B

(3) Complement

$x \in A^c$ iff $x \notin A$

$$A^c = S - A$$

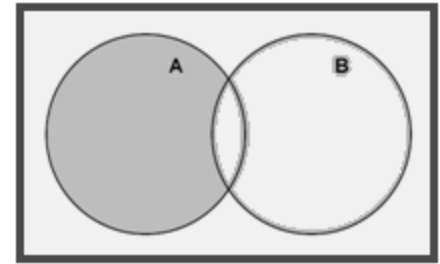


□ Operations in Set Algebra

(4) Difference

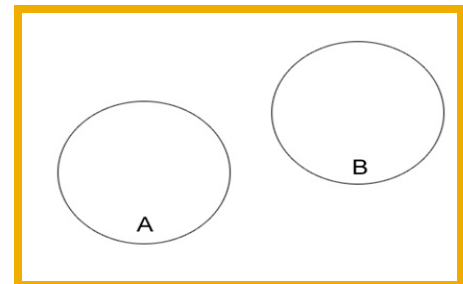
$x \in A - B$ iff $x \in A$ & $x \notin B$

$$A - B = A \cap B^c$$



disjoint

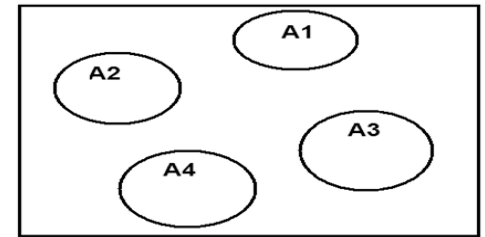
A and B called disjoint iff $A \cap B = \Phi$



Mutually Exclusive متنافية

A collection of sets $A_1, \dots, A_n$ is mutually exclusive iff

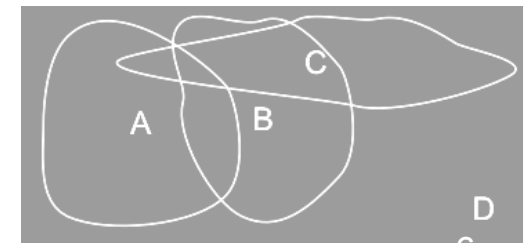
$$A_i \cap A_j = \Phi$$



Collectively Exhaustive مجمعة شاملة

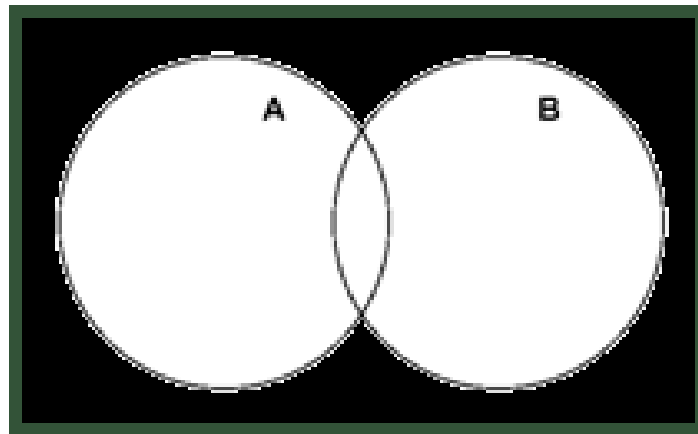
A collection of sets $A_1, \dots, A_n$ is collectively exhaustive iff

$$A_1 \cup A_2 \cup A_3 \dots = S$$



DEMORGAN LAW

$$(A \cup B)^c = A^c \cap B^c$$



Quiz 1.1

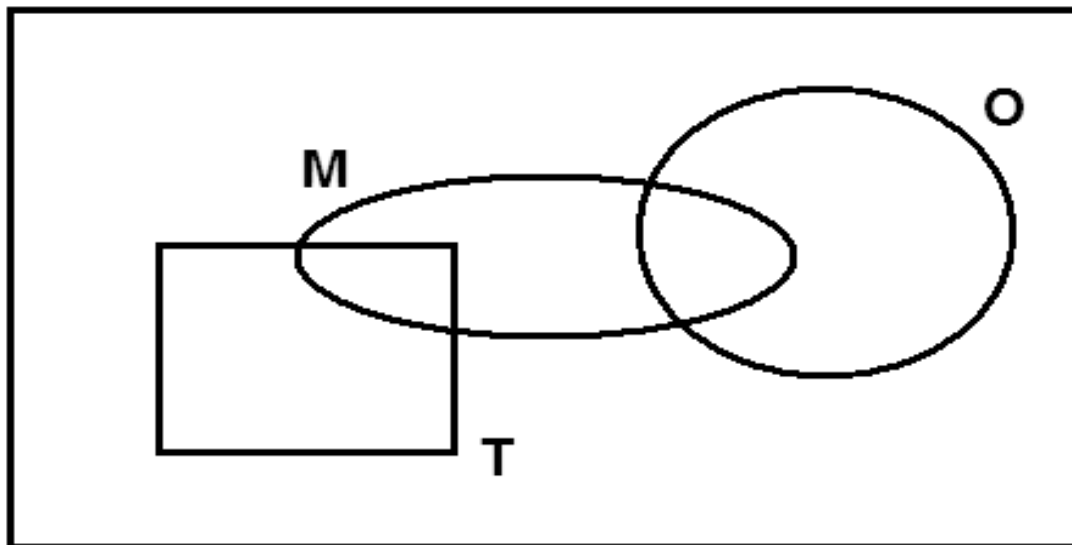
A slice of pizza sold by Gerlanda's Pizza is either

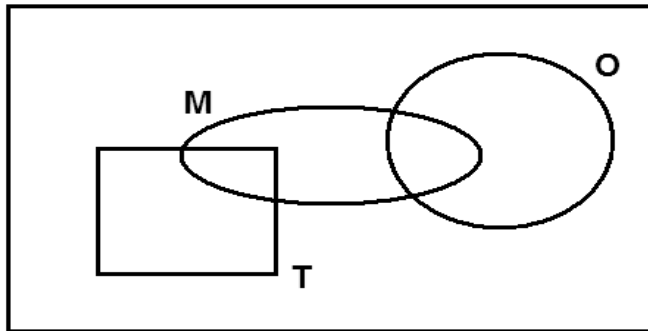
Regular Pizza(R) or **Tuscan(T)**

Each slice may have mushrooms(M) or onions(O)

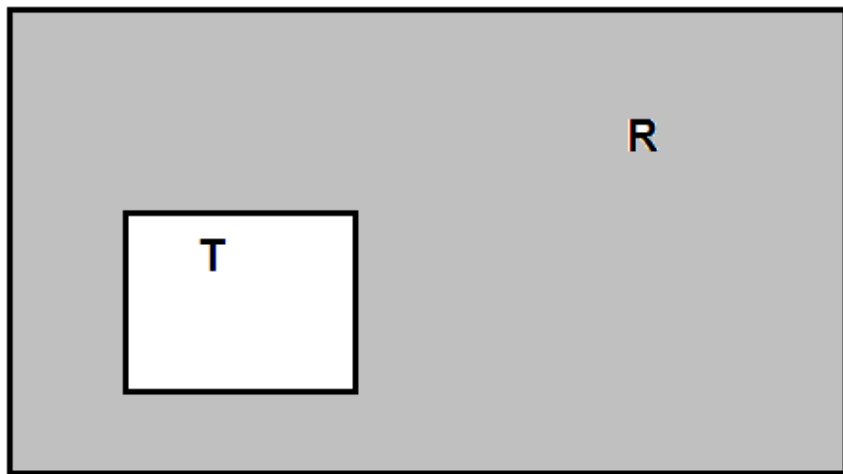
As described by the Venn diagram.

Shade the region of Venn diagram for the following sets:

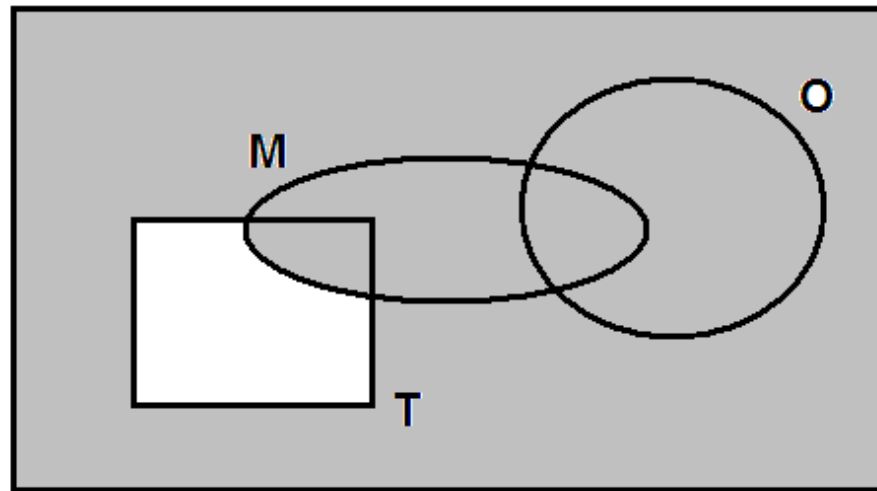




(a) R

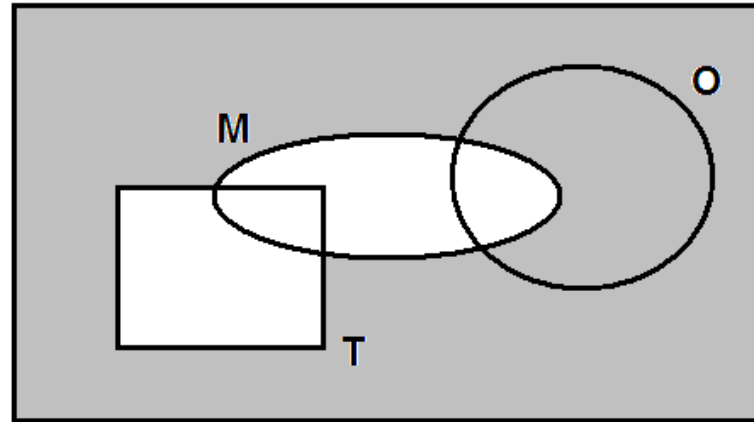


(d) $R \cup M$

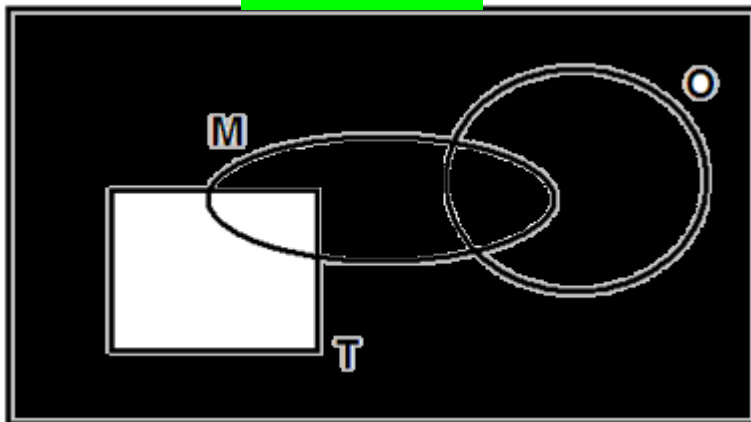


$$(f) T^c - M$$

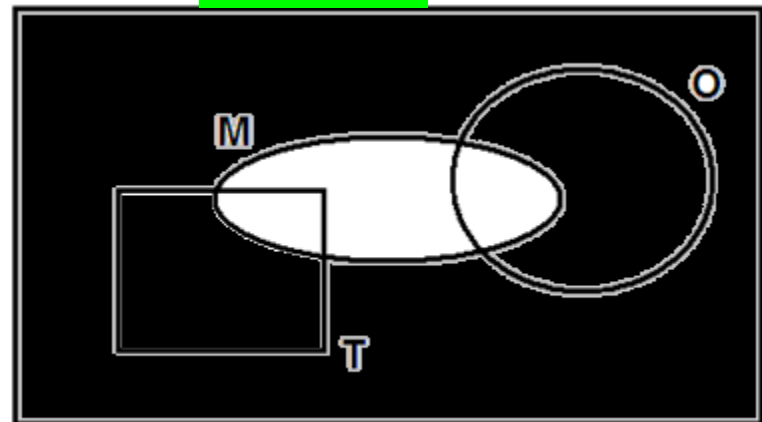
$$= T^c \cap M^c$$



T^c



M^c

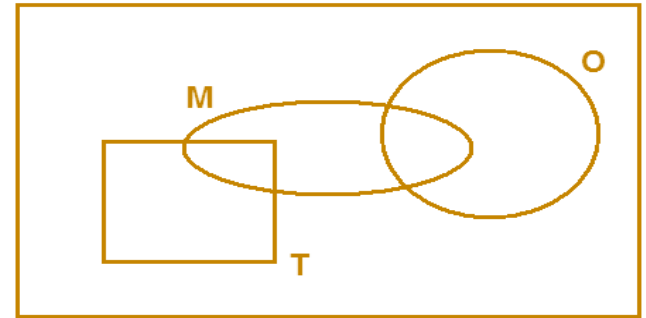


Problem 1.1.1

From Quiz1.1 ask the following questions:

(a) Are T and M mutually exclusive?

No, $T \cap M \neq \Phi$



(b) Are R,T and M collectively exhaustive ?

Yes, $R \cup T \cup M = S$

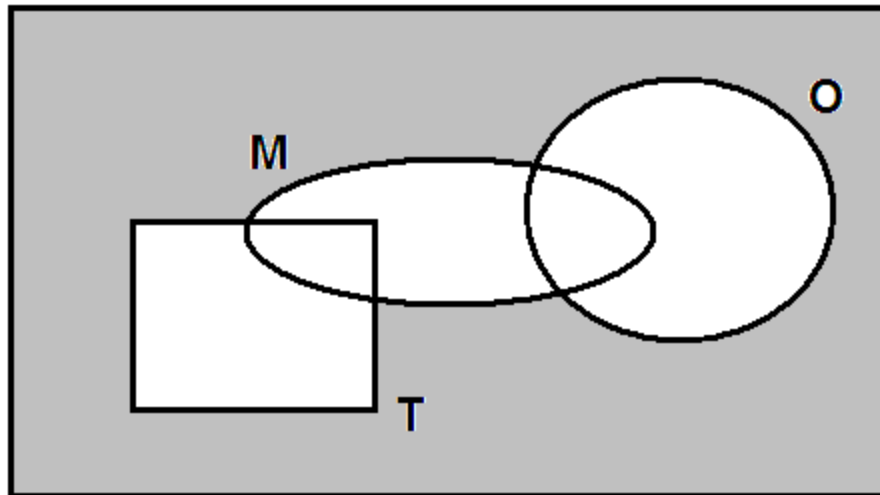
(d) Does Gerlands make T pizzas with M and O?

No, T & O are Mutually exclusive

Problem 1.1.1

(e) Does Gerlands make R pizzas that have neither M nor O ?

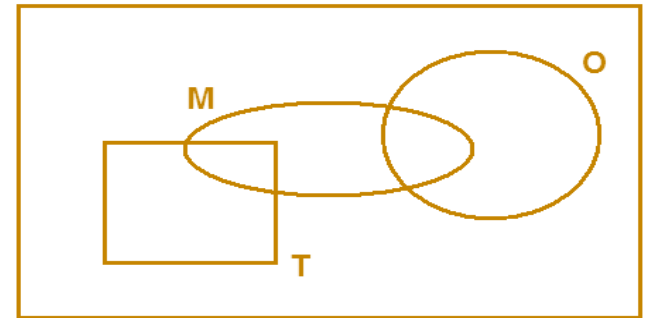
Yes, as shown in Venn diagram



Problem 1.1.2

From Quiz1.1 write Gerlanda's menu in words:

- Regular with mushrooms and onions
- Tuscan with mushrooms
- Tuscan without mushrooms
-



1.2 Applying Set Theory to Probability

Sample Space

فضاء العينة

Sample space of an experiment is the:

- **finest grain** (all possible outcomes identified separately)
- **mutually exclusive and**
- **collectively exhaustive set.**

e.g.: Roll six-sided die one time
 $S = \{1, 2, 3, 4, 5, 6\}$

Event

الحدث

Subset of sample space

e.g.: event $E1 = \{\text{Roll 4 or higher}\} = \{4, 5, 6\}$

Note: event that contains all sample space is called event space

Quiz 1.2

Letter may be: **voice call(v)** or **data call(d)**

Observation a sequence of three letters .

Write the elements of the following sets.

(1) A1=First call is a voice call

$A1=\{vvv, vvd, vdv, vdd\}$

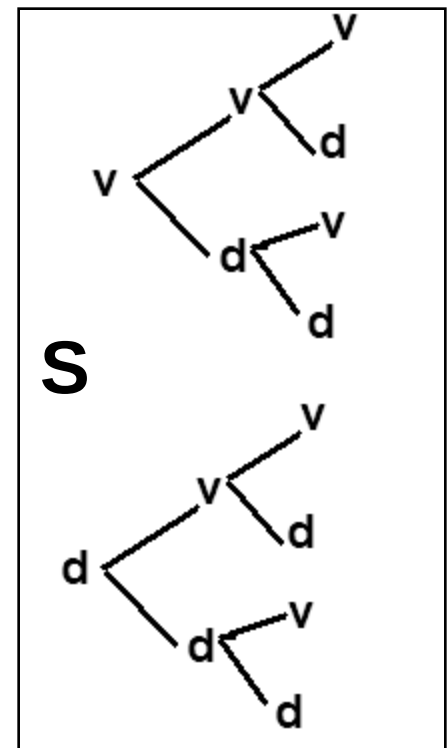
(3) A3=All call are the same

$A1=\{vvv, ddd\}$

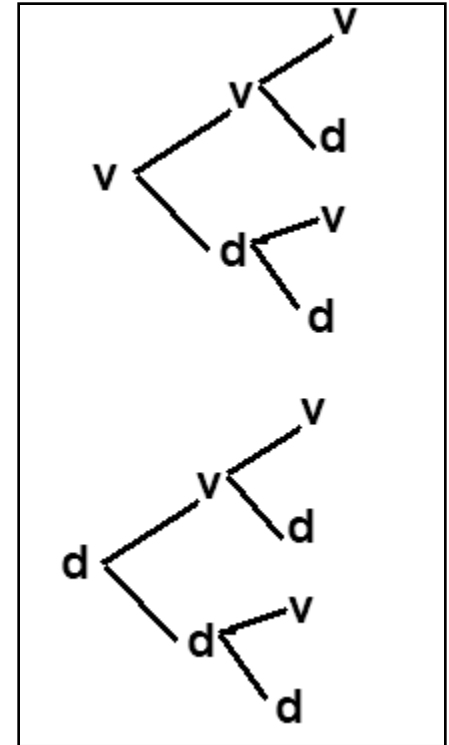
(5) B1=First call is data call

$A1=\{dvv, dvd, ddv, ddd\}$

A1&B1 are Mutually exclusive and collectively exhaustive



(7) Voice and data alternate
 $A1 = \{vdv, dvd\}$



Problem 1.2.1

Experiment of monitoring a fax transmission

Speed: low(l),medium(m),high(h)

Length: short faxes 2p(t),long faxes 4p(f)

(a) What is the sample space of the experiment ?

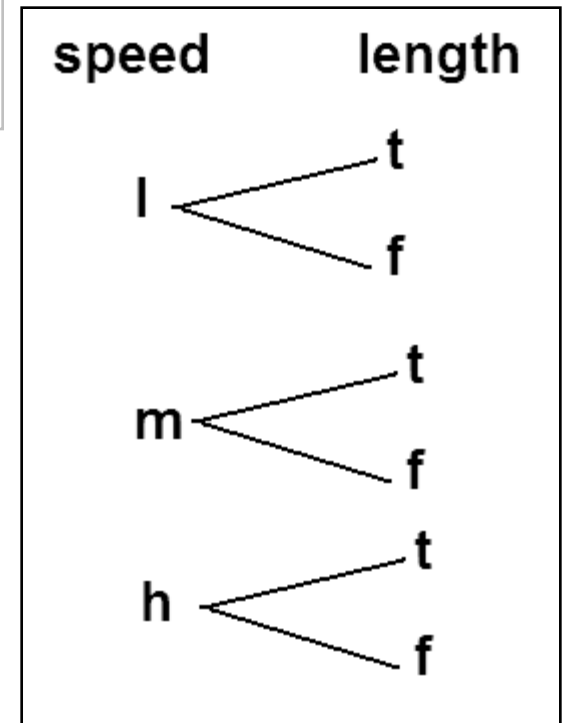
$$S.S = \{lt, lf, mt, mf, ht, hf\}$$

(b) Event $A1=\{\text{medium speed fax}\}$

$$A1 = \{mt, mf\}$$

(c) Event $A2=\{\text{short fax 2p.}\}$

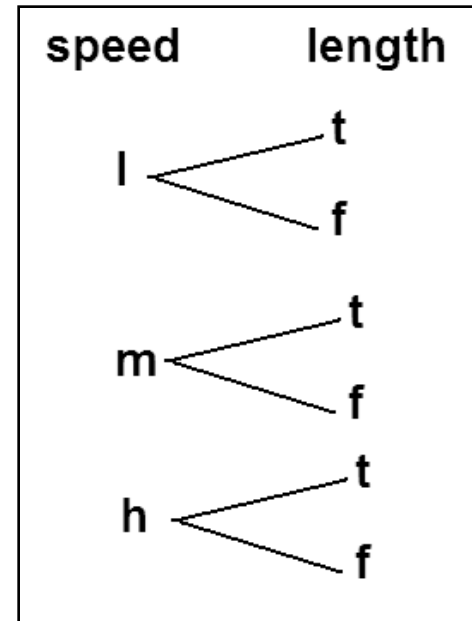
$$A2 = \{lt, mt, ht\}$$



Problem 1.2.1

(d) Event $A3$ = High speed h or low speed ?

$$A3 = \{ht, hf, lt, lf\}$$



(e) Are $A1, A2$ and $A3$ Mutually exclusive

$$\text{No, } A1 \cap A2 = \{mt\} \neq \Phi$$

(f) Are $A1, A2$ and $A3$ collectively exhaustive

$$\text{Yes, } A1 \cup A2 \cup A3 = \{S\}$$

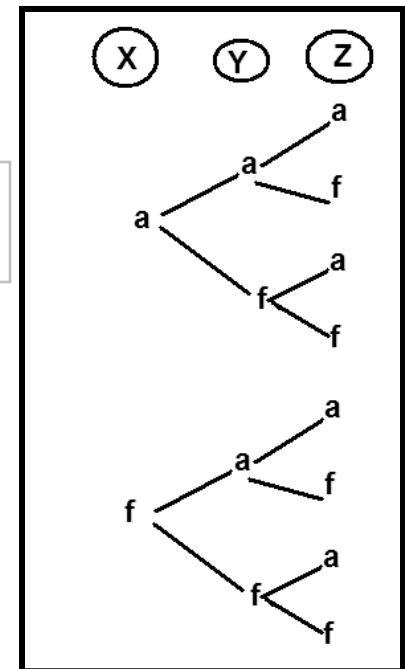
Problem 1.2.2

Three machines X,Y and Z are produced **integrated circuits** :**acceptable(a)**,**fails(f)**

Observe a sequence of **three test** results from the three machines ?

(a) Sample space

$S.S = \{aaa, aaf, afa, aff, faa, faf, fffa, fff\}$



1.3,1.4 Probability Axioms

$$(1) P[A] \geq 0$$

$$(2) P[S] = 1$$

$$(3) P[A \cup B] = P[A] + P[B] - P[A \cap B]$$

If A&B Mutually exclusive
Then: $P[A \cup B] = P[A] + P[B]$

$$A1 \cap A2 = \Phi$$

$$(4) P[\Phi] = 0$$

$$(5) P[A^c] = 1 - P[A]$$



$$(6) P[A \cap B] = P[A, B] = P[AB], \text{ A and B}$$

$$(7) P[A \cap A] = P[A.A] = P[A]$$

○ **Equally likely Outcomes:**

$$P[Si] = \frac{1}{n}$$

e.g. Roll six-sided die in which all faces are equally likely

$E1 = \{\text{roll 4 or higher}\} = \{4, 5, 6\}$

$P[E1] = 3/6 = 1/2$



Quiz 1.3

Test grades (T) 0-100

Score 90-100 (A) , 80-89 (B) , 70-79 (c) , 60-69 (D) ,less 60 (F)

All scores between 51-100 (S.S=50 elements)

All scores are equally likely

Find the following probability :

$$(a) P[\{S_{79}\}] = \frac{1}{50} \quad (d) P[T \geq 80] = \frac{21}{50} = 0.42$$

$$(c) P[A] = \frac{11}{50} \quad (e) P[T < 90] = \frac{39}{50} = 0.78$$

$$(f) P[C \text{ grade or better}] = (70 \rightarrow 100) = \frac{31}{50} = 0.62$$

$$(g) P[pass] = (60 \rightarrow 100) = \frac{41}{50} = 0.82$$

Quiz 1.4

Classify the call as:

Long(L) or Brief(B)

Voice(V) or Data(D)

$$P[V]=0.7, P[L]=0.6, P[VL]=0.35$$

Find the following probabilities:

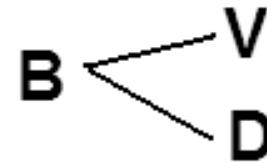
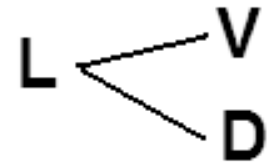
$$P[V]=P[LV]+P[VB] \rightarrow P[VB]=0.7-0.35=0.35$$

$$P[L]=P[LV]+P[LD] \rightarrow P[LD]=0.6-0.35=0.25$$

$$P[LV]+P[LD]+P[BV]+P[BD]=P[S]=1$$

$$\rightarrow P[BD]=1-(0.25+0.35+0.35)=0.05$$

$$S.S = \{LV, LD, BV, BD\}$$



	V	D	
L	0.35	0.25	0.6
B	0.35	0.5	0.4
	0.7	0.3	1

$$(1) P[DL] = 0.25$$

	V	D	
L	0.35	0.25	0.6
B	0.35	0.5	0.4
	0.7	0.3	1

$$(2) P[D \cup L] = P[D] + P[L] - P[D \cap L]$$

$$= 0.3 + 0.6 - 0.25 = 0.65$$

$$(6) P[LB] = 0$$

Problem 1.3.1

Computer program are classified by:
Length of the code & **Execution time**

More than 150 lines → **Big (B)**

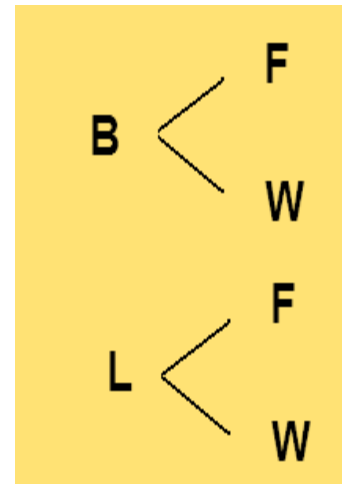
Less than or equal 150 lines → **Little(L)**

program run in less than 0.1 sec → **Fast(F)**

program required at least 0.1 sec → **Slow(W)**

$P[LF]=0.5$, $P[BF]=0.2$, $P[BW]=0.2$

What is the Sample Space(S.S) ?



$S.S = \{BF, BW, LF, LW\}$

	B	L
F	0.2	0.5
W	0.2	0.1

Problem 1.3.1

Calculate:

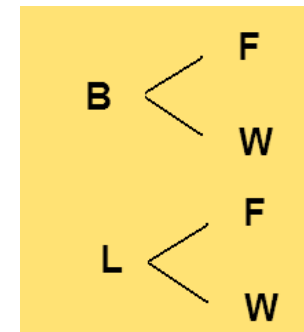
$$(a) P[W] = 0.2 + 0.1 = 0.3$$

$$(b) P[B] = 0.2 + 0.2 = 0.4$$

$$(c) P[W \cup B] = P[W] + P[B] - P[W \cap B] = 0.3 + 0.4 - 0.2 = 0.5$$

$$S.S = \{BF, BW, LF, LW\}$$

	B	L	
F	0.2	0.5	0.7
W	0.2	0.1	0.3
	0.4	0.6	



Problem 1.3.4

Roll six-sided die once ,observe the number of dots.

What is the sample space?

$$S.S = \{1,2,3,4,5,6\}$$



What is the probability of each sample outcome ?

$$P[S_i] = \frac{1}{6}$$

What is the probability of E, the event that the roll is even ?

$$E = \{2,4,6\}$$

$$P[E] = P[\{2\}] + P[\{4\}] + P[\{6\}] = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$$

Problem 1.3.5

Quiz of 10 point

Score of students between 0 → 10

- What is the probability of A, which requires the student to get a score of 9 or more ?

$$A = \{9, 10\}$$

$$P[A] = P[\{9\}] + P[\{10\}] = \frac{1}{11} + \frac{1}{11} = \frac{2}{11}$$

- What is the probability of F, which requires the student to get a score of less than 4?

$$A = \{0, 1, 2, 3\} \quad , \quad P[A] = \frac{4}{11}$$

Problem 1.3.5

Quiz of 10 point

Score of students between 0 → 10

- What is the probability of A, which requires the student to get a score of 9 or more ?

$$A = \{9, 10\}$$

$$P[A] = P[\{9\}] + P[\{10\}] = \frac{1}{11} + \frac{1}{11} = \frac{2}{11}$$

- What is the probability of F, which requires the student to get a score of less than 4?

$$A = \{0, 1, 2, 3\} \quad , \quad P[A] = \frac{4}{11}$$

Problem 1.4.1

Mobile telephones perform *handoffs* as they move from cell to cell. During a call, a telephone either performs zero handoffs (H_0), one handoff (H_1), or more than one handoff (H_2). In addition, each call is either long (L), if it lasts more than three minutes, or brief (B). The following table describes the probabilities of the possible types of calls.

	H_0	H_1	H_2
L	0.1	0.1	0.2
B	0.4	0.1	0.1

What is the probability $P[H_0]$ that a phone makes no handoffs? What is the probability a call is brief? What is the probability a call is long or there are at least two handoffs?

(a) Find $P[H_0]$?

$$P[H_0] = 0.1 + 0.4 = 0.5$$

(b) Find $P[B]$?

$$P[B] = 0.4 + 0.1 + 0.1 = 0.6$$

(c) Find $P[L \cup H_2]$?

$$= P[L] + P[H_2] - P[L \cdot H_2] = 0.4 + 0.3 - 0.2 = 0.5$$

	H₀	H₁	H₂	
L	0.1	0.1	0.2	0.4
B	0.4	0.1	0.1	0.6
	0.5	0.2	0.3	1

Thank You !